

# Supplemental file of MOMMOP

## Figure Caption

- Fig. S1. The average number of optimal solutions found by MOMMOP during the evolution.

## Table Captions

- Table S1 Peak ratio (PR) and success rate (SR) of MOMMOP
- Table S2 Comparison of MOMMOP with respect to dADE/nrand/1, NVMO, PNSGA-II, and MNSGA-II in terms of the peak ratio (PR) with  $\varepsilon = 1.0E - 04$ . The best PR value is highlighted in boldface for each test function.
- Table S3 Comparison of MOMMOP with respect to NCDE, NSDE, LIPS, and r2pso in terms of the peak ratio (PR) with  $\varepsilon = 1.0E - 04$ . The best PR value is highlighted in boldface for each test function.
- Table S4 Comparison of MOMMOP with respect to MOBiDE and BMPGA in terms of the peak ratio (PR). The best PR value is highlighted in boldface for each test function.
- Table S5 Comparison of MOMMOP with respect to MOMMOP1 and MOMMOP2 in terms of the peak ratio (PR) with  $\varepsilon = 1.0E - 04$ . The best PR value is highlighted in boldface for each test function.
- Table S6 Comparison of MOMMOP with varying distance parameter in terms of the peak ratio (PR) with  $\varepsilon = 1.0E - 04$ . The best PR value is highlighted in boldface for each test function.
- Table S7 Comparison of MOMMOP with varying scaling parameter in terms of the peak ratio (PR) with  $\varepsilon = 1.0E - 04$ . The best PR value is highlighted in boldface for each test function.

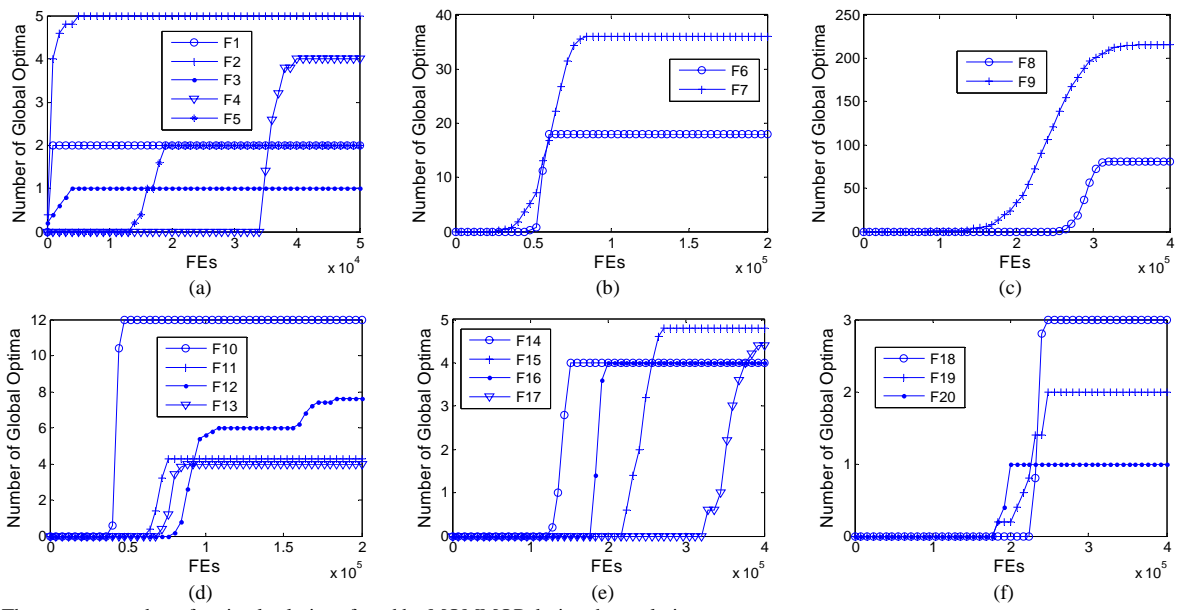


Fig. S1. The average number of optimal solutions found by MOMMOP during the evolution.

TABLE S1  
PEAK RATIO (PR) AND SUCCESS RATE (SR) OF MOMMOP

Accuracy Level $\varepsilon$	F1		F2		F3		F4		F5	
	PR	SR	PR	SR	PR	SR	PR	SR	PR	SR
1.0E-01	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.0E-02	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.0E-03	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.0E-04	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.0E-05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Accuracy Level $\varepsilon$	F6		F7		F8		F9		F10	
	PR	SR	PR	SR	PR	SR	PR	SR	PR	SR
1.0E-01	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.0E-02	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.0E-03	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.0E-04	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.940	1.000	1.000
1.0E-05	1.000	1.000	1.000	1.000	1.000	1.000	0.977	0.100	1.000	1.000
Accuracy Level $\varepsilon$	F11		F12		F13		F14		F15	
	PR	SR	PR	SR	PR	SR	PR	SR	PR	SR
1.0E-01	1.000	1.000	0.995	0.960	0.960	0.780	0.783	0.000	0.675	0.000
1.0E-02	0.990	0.940	0.985	0.880	0.933	0.640	0.727	0.000	0.645	0.000
1.0E-03	0.940	0.640	0.965	0.740	0.667	0.000	0.667	0.000	0.623	0.000
1.0E-04	0.717	0.020	0.960	0.700	0.667	0.000	0.667	0.000	0.605	0.000
1.0E-05	0.670	0.000	0.840	0.120	0.667	0.000	0.667	0.000	0.588	0.000
Accuracy Level $\varepsilon$	F16		F17		F18		F19		F20	
	PR	SR	PR	SR	PR	SR	PR	SR	PR	SR
1.0E-01	0.667	0.000	0.528	0.000	0.500	0.000	0.250	0.000	0.125	0.000
1.0E-02	0.667	0.000	0.528	0.000	0.500	0.000	0.250	0.000	0.125	0.000
1.0E-03	0.667	0.000	0.528	0.000	0.500	0.000	0.250	0.000	0.125	0.000
1.0E-04	0.667	0.000	0.518	0.000	0.500	0.000	0.250	0.000	0.125	0.000
1.0E-05	0.667	0.000	0.490	0.000	0.500	0.000	0.250	0.000	0.125	0.000

TABLE S2  
 COMPARISON OF MOMMOP WITH RESPECT TO dADE/NRAND/1, NVMO, PNSGA-II, AND MNSGA-II IN TERMS OF THE PEAK RATIO (PR)  
 WITH  $\varepsilon = 1.0E - 04$ . THE BEST PR VALUE IS HIGHLIGHTED IN BOLDFACE FOR EACH TEST FUNCTION.

Test Function	dADE/nrand/1	NVMO	PNSGA-II	MNSGA-II	MOMMOP
F1	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	0.930	<b>1.000</b>
F2	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F3	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F4	<b>1.000</b>	<b>1.000</b>	0.985	0.320	<b>1.000</b>
F5	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	0.900	<b>1.000</b>
F6	0.984	0.670	0.473	0.001	<b>1.000</b>
F7	0.823	0.901	0.709	0.509	<b>1.000</b>
F8	0.967	0.198	0.275	0.000	<b>1.000</b>
F9	0.431	0.275	0.298	0.140	<b>1.000</b>
F10	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	0.953	<b>1.000</b>
F11	0.667	0.667	0.680	0.033	<b>0.717</b>
F12	0.740	0.713	0.642	0.010	<b>0.960</b>
F13	<b>0.667</b>	<b>0.667</b>	0.663	0.000	<b>0.667</b>
F14	<b>0.667</b>	<b>0.667</b>	0.663	0.000	<b>0.667</b>
F15	<b>0.627</b>	0.623	0.470	0.000	0.605
F16	<b>0.667</b>	0.653	0.417	0.000	<b>0.667</b>
F17	0.403	0.413	0.300	0.000	<b>0.518</b>
F18	<b>0.633</b>	0.470	0.110	0.000	0.500
F19	0.018	0.130	0.017	0.000	<b>0.250</b>
F20	0.005	0.000	0.000	0.000	<b>0.125</b>

TABLE S3

COMPARISON OF MOMMOP WITH RESPECT TO NCDE, NSDE, LIPS, AND R2PSO IN TERMS OF THE PEAK RATIO (PR) WITH  $\varepsilon = 1.0E - 04$ . THE BEST PR VALUE IS HIGHLIGHTED IN BOLDFACE FOR EACH TEST FUNCTION.

Test Function	NCDE	NSDE	LIPS	r2pso	MOMMOP
F1	<b>1.000</b>	<b>1.000</b>	0.000	0.000	<b>1.000</b>
F2	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	0.992	<b>1.000</b>
F3	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F4	<b>1.000</b>	0.995	<b>1.000</b>	0.670	<b>1.000</b>
F5	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F6	0.972	0.993	0.998	0.388	<b>1.000</b>
F7	0.917	0.685	0.527	0.509	<b>1.000</b>
F8	0.062	0.793	0.876	0.000	<b>1.000</b>
F9	0.668	0.384	0.268	0.091	<b>1.000</b>
F10	0.998	0.981	0.987	0.788	<b>1.000</b>
F11	0.667	0.990	<b>0.993</b>	0.667	0.717
F12	0.053	<b>0.990</b>	0.960	0.448	0.960
F13	0.667	0.667	<b>0.770</b>	0.660	0.667
F14	<b>0.667</b>	<b>0.667</b>	<b>0.667</b>	0.003	<b>0.667</b>
F15	0.373	0.497	0.590	0.003	<b>0.605</b>
F16	0.663	<b>0.667</b>	<b>0.667</b>	0.000	<b>0.667</b>
F17	0.250	0.275	0.498	0.000	<b>0.518</b>
F18	0.357	<b>0.547</b>	0.500	0.000	0.500
F19	0.020	0.243	<b>0.250</b>	0.000	<b>0.250</b>
F20	0.000	0.000	0.000	0.000	<b>0.125</b>

TABLE S4  
 COMPARISON OF MOMMOP WITH RESPECT TO MOBIDE AND BMPGA IN TERMS OF THE PEAK RATIO (PR). THE BEST PR VALUE IS HIGHLIGHTED IN BOLDFACE FOR EACH TEST FUNCTION.

Test Function	The niche radius $r$	$\varepsilon$	MOBiDE	BMPGA	MOMMOP
F1	0.5	1E-06	<b>1.000</b>	0.750	<b>1.000</b>
F2	0.01	1E-06	<b>1.000</b>	0.928	<b>1.000</b>
F3	0.01	1E-06	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F4	0.5	5E-04	<b>1.000</b>	0.835	<b>1.000</b>
F5	0.5	1E-05	<b>1.000</b>	0.620	<b>1.000</b>
F6	0.2	5E-02	0.967	0.762	<b>1.000</b>
F7	0.2	1E-03	0.983	0.882	<b>1.000</b>
F9	0.1	1E-03	0.814	0.563	<b>1.000</b>

TABLE S5

COMPARISON OF MOMMOP WITH RESPECT TO MOMMOP1 AND MOMMOP2 IN TERMS OF THE PEAK RATIO (PR) WITH  $\varepsilon = 1.0E - 04$ . THE BEST PR VALUE IS HIGHLIGHTED IN BOLDFACE FOR EACH TEST FUNCTION.

Test Function	MOMMOP1	MOMMOP2	MOMMOP
F1	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F2	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F3	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F4	<b>1.000</b>	0.015	<b>1.000</b>
F5	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F6	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F7	0.853	0.994	<b>1.000</b>
F8	0.422	0.887	<b>1.000</b>
F9	0.509	0.961	<b>1.000</b>
F10	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F11	0.680	0.567	<b>0.717</b>
F12	0.865	0.570	<b>0.960</b>
F13	<b>0.667</b>	0.470	<b>0.667</b>
F14	<b>0.667</b>	0.060	<b>0.667</b>
F15	0.478	0.015	<b>0.605</b>
F16	<b>0.667</b>	0.006	<b>0.667</b>
F17	0.125	0.192	<b>0.518</b>
F18	0.180	0.000	<b>0.500</b>
F19	0.125	0.010	<b>0.250</b>
F20	<b>0.125</b>	<b>0.125</b>	<b>0.125</b>

TABLE S6  
 COMPARISON OF MOMMOP WITH VARYING DISTANCE PARAMETER IN TERMS OF THE PEAK RATIO (PR) WITH  $\varepsilon = 1.0E - 04$ . THE BEST PR VALUE IS HIGHLIGHTED IN BOLDFACE FOR EACH TEST FUNCTION.

Test Function	0.001 (MOMMOP_dp1)	0.005 (MOMMOP_dp2)	0.01 (MOMMOP_dp3)	0.05 (MOMMOP_dp4)
F1	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F2	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F3	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	0.903
F4	0.305	<b>1.000</b>	<b>1.000</b>	0.975
F5	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F6	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	0.413
F7	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	0.690
F8	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	0.333
F9	0.998	0.999	<b>1.000</b>	0.713
F10	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F11	0.696	0.710	<b>0.717</b>	0.706
F12	0.952	0.948	<b>0.960</b>	0.932
F13	0.667	<b>0.670</b>	0.667	0.667
F14	<b>0.667</b>	<b>0.667</b>	<b>0.667</b>	<b>0.667</b>
F15	0.557	<b>0.640</b>	0.605	0.602
F16	<b>0.667</b>	<b>0.667</b>	<b>0.667</b>	<b>0.667</b>
F17	0.412	0.490	<b>0.518</b>	0.395
F18	0.453	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>
F19	0.167	0.250	0.250	<b>0.255</b>
F20	<b>0.125</b>	<b>0.125</b>	<b>0.125</b>	<b>0.125</b>



TABLE S7  
 COMPARISON OF MOMMOP WITH VARYING SCALING PARAMETER IN TERMS OF THE PEAK RATIO (PR) WITH  $\varepsilon = 1.0E - 04$ . THE BEST PR VALUE IS HIGHLIGHTED IN BOLDFACE FOR EACH TEST FUNCTION.

Test Function	10 (MOMMOP_sp1)	30 (MOMMOP_sp2)	40 (MOMMOP_sp3)	50 (MOMMOP_sp4)	80 (MOMMOP_sp5)
F1	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F2	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F3	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F4	0.015	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	0.970
F5	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	0.986
F6	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F7	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F8	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F9	0.983	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F10	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>
F11	0.690	0.707	<b>0.717</b>	<b>0.717</b>	0.713
F12	0.845	0.940	<b>0.960</b>	0.935	0.930
F13	0.660	0.657	<b>0.667</b>	0.657	0.645
F14	<b>0.667</b>	<b>0.667</b>	<b>0.667</b>	<b>0.667</b>	<b>0.667</b>
F15	0.462	0.598	<b>0.605</b>	<b>0.605</b>	0.582
F16	<b>0.667</b>	<b>0.667</b>	<b>0.667</b>	<b>0.667</b>	0.633
F17	0.015	0.502	<b>0.518</b>	0.497	0.475
F18	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>	0.483
F19	<b>0.250</b>	<b>0.250</b>	<b>0.250</b>	<b>0.250</b>	0.185
F20	<b>0.125</b>	<b>0.125</b>	<b>0.125</b>	<b>0.125</b>	<b>0.125</b>